Multifractal Measures for the Yen-Dollar Exchange Rate

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We study the tick dynamical behavior of the yen-dollar exchange rate using the rescaled range analysis in financial market. It is found that the multifractal Hurst exponents with the short and long-run memory effects can be obtained from the yen-dollar exchange rate. This exists one crossover for the Hurst exponents at charateristic time scales, while the bond futures exists no crossover. Particularly, it is shown that the probability distribution of the yen-dollar exchange rate has one form of the Lorentz distribution rather than fat-tailed properties, which is similar to that of for the won-dollar exchange rate.

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Recently, the outstanding topics in econophysics have mainly included the price changes in open market [1, 2], the distribution of income of companies, the scaling relation of size fluctuations of companies, the financial analysis of foreign exchange rates [3], the tick data analysis of bond futures [4], the herd behavior of financial markets [5], the self-organized segregation, and the minority-majority game [6]. In particular, the essential problems with fluctuations have particularly led to a better understanding for the scaling properties based on methods and approaches in scientific fields. It was discussed in the previous work [3] that the price fluctuations follow the anomalous power law from the stochastic time evolution equation, which is represented in terms of the Langevin-type equation. Furthermore, the power law distribution, the stretched exponential distribution, and the fat-tailed distribution have showed the functional properties from the numerical results obtained in diverse econophysical systems.

To measure the multifractals of dynamical dissipative systems, the generalized dimension and the spectrum have effectively used to calculate the trajectory of chaotic attractors that may be classified by the type and number of the unstable periodic orbits. Many attempts [7 − 9] to compute these statistical quantities have primarily presented from the box-counting method. We have used the box-counting method to analyze precisely generalized dimensions and scaling exponents for the mountain height and the sea-bottom depth [10]. For the standard analysis, since there exists notably no statistical correlations between observations, the R/S analysis[11] has extended to distinguish the random time series from correlated ones.

Scalas *et al*. [4] have studied the correlation function for bond walks from the time series of Buoni del tesoro Poliennali futures exchanged at the London International Financial Futures and options Exchange(LIFFE). They have discussed that the continuous-time random walk theory [12] is sucessfully applied to the dynamical behavior of empirical scaling laws by a set of tick-by-tick data in financial markets. Mainardi *et al*. [13] have also argued on the waiting-time distribution for bond futures traded at LIFFE. The theoretical and numerical arguments for the volume of bond futures were presented at Korean Futures Exchange market [14]. The

studies of multifractals in financial markets have explored up to now, but it is of fundamental importance to treat with the multifractal nature of prices for the yen-dollar exchange rate. In this paper, we treat with the generic multifractal behavior for tick data of prices using the R/S analysis for the yen-dollar exchange rate. The result obtained is also compared with that of the won-dollar exchange rate, and the multifractal Hurst exponents, the price-price correlation function, and the probability distribution of returns are particularly discussed with long-run memory effects. Our result obtained is also compared with that of the won-dollar exchange rate.

To quantify the Hurst exponents, we introduce the R/S analysis method that is generally contributed to estimate the multifractals [15, 16]. At first we suppose a price time series of length n given by $\{p(t_1), p(t_2), ..., p(t_n)\}$, and the price τ-returns $r(\tau)$ having time scale τ and length n that is represented in terms of $r(\tau) = \{r_1(\tau), r_2(\tau), ..., r_n(\tau)\}$, with $r_i(\tau) = \ln p(t_i+\tau) - \ln p(t_i)$. For simplicity, after dividing the time series or returns into N subseries of length M, we label each subseries $E_{M,d}(\tau) = \{r_{1,d}(\tau), r_{2,d}(\tau), ..., r_{M,d}(\tau)\}$, with $d = 1, 2, ..., N$. Then, the deviation $D_{M,d}(\tau)$, i.e., the differences between $r_{M,d}(\tau)$ and $\bar{r}_{M,d}(\tau)$ for all M, can be defined from the mean of returns $\bar{r}_{M,d}(\tau)$ as

$$D_{M,d}(\tau) = \sum_{k=1}^{M} (r_{k,d}(\tau) - \bar{r}_{M,d}(\tau)). \tag{1}$$

The hierachical average value $(R/S)_M(\tau)$ represented the rescaled and normalized relation between the subseries $R_{M,d}(\tau)$ and the standard deviation $S_{M,d}(\tau)$ becomes

$$(R/S)_M(\tau) = \frac{1}{N} \sum_{d=1}^{N} \frac{R_{M,d}(\tau)}{S_{M,d}(\tau)} \propto M^{H(\tau)}, \tag{2}$$

where $H(\tau)$ is called the Hurst exponent, and the statistical quantities $R_{M,d}(\tau)$ and $S_{M,d}(\tau)$ are, respectively, given by

$$\begin{aligned} R_{M,d}(\tau) &= max\{D_{1,d}(\tau), D_{2,d}(\tau), ..., D_{M,d}(\tau)\} \\ &\quad -min\{D_{1,d}(\tau), D_{2,d}(\tau), ..., D_{M,d}(\tau)\} \end{aligned} \tag{3}$$

and

$$S_{M,d}(\tau) = [\frac{1}{M}\sum_{k=1}^{M}(r_{k,d}(\tau) - \bar{r}_{M,d}(\tau))^2]^{\frac{1}{2}}. \quad (4)$$

For more than one decade, several methods have suggested in order to investigate the multifractal properties systematically. From the multifractality of self-affine fractals $[7-9]$, the q-th price-price correlation function $F_q(\tau)$ takes the form

$$F_q(\tau) =< |p(t+\tau) - p(t)|^q >\propto \tau^{qH_q}, \quad (5)$$

where τ is the time lag, H_q is the generalized qth-order Hurst exponent, and the angular brackets denote a statistical average over time. When our simulation is performed on the price $p(t)$, a nontrivial multi-affine spectrum can be obtained as H_q varies with q. This has exploited in the multifractal method and the large fluctuation effects in the dynamical behavior of the price can be explored from Eq.(5). In our scheme, we will make use of Eqs.(2) and (5) to compute the multifractal features of prices, and the mathematical techniques discussed lead us to more general results.

For characteristic analysis of the yen-dollar exchange rate, we will present in detail numerical data of Hurst exponents from the results of R/S analysis. Although we extend to find other statistical quantities via computer simulation studies of returns, we resrict ourselves to estimate the generalized qth-order Hurst exponents in the price-price correlation function and the form of the probability distribution of returns. In this paper, we introduce the price time series for the yen-dollar exchange rate, in which the time step between ticks is evoluted for one day and the tick data for the yen-dollar exchange rate were taken from January 1971 to June 2003. The Hurst exponents are obtained numerically from the results of R/S analysis given by Eq.(2), as summarized in Table 1. Fig.1 shows the tick data of returns $R(t)$ as a function $\tau = 1$(one day) for the yen-dollar exchange rate, and the Hurst exponents for the yen-dollar exchange rate are $H(\tau = 1)$=0.6513, as plotted in Fig.2. It is found that our Hurst values are significantly different from the random walk with $H = 0.5$, which this process are located in the persistence region similar to those of the crude oil prices

[16]. Especially, it may be expected that the Hurst exponent is taken anomalously to be near 1 as the time series proceeds with long-run memory effects. The crossover in the Hurst exponent $H(\tau)$ is not existed, while $H(\tau)$ from our tick data is similarly found to have the existence of crossovers at characteristc time $\tau = 9(\tau = 7$ and $35)$ for the won-dollar exchange rate(the KOSPI) [17].

For the sake of concreteness, we perform the numerical study of Eq.(5) in order to analyze the generalized qth-order Hurst exponents in the price-price correlation function $F_q(\tau)$. Table 1 includes the values of the generalized qth-order Hurst exponent H_q in the price-price correlation function for the yen-dollar exchange rate. Especially, the generalized Hurst exponent is taken to be near 0.6216 as $q \to 1$, and the values $\log(F_q/q)$ for $q = 1, 2, ..., 6$ are plotted in Fig.3 for the the yen-dollar exchange rate. The probability distribution of returns is well consistent with a Lorentz distribution different from fat-tailed properties, as shown in Figs.4 and 5.

In summary, we have presented the multifractal measures from the dynamical behavior of prices using the R/S analysis for the yen-dollar exchange rate. The multifractal Hurst exponents, the generalized qth-order Hurst exponent, and the form of the probability distribution have discussed with long-run memory effects. Since Hurst exponents are larger than 0.5 through R/S analysis, our case for time series of prices is the persistent process. It is apparent from our data of the Hurst exponent $H(\tau)$ that the existence of crossovers is similar to that of other result [16]. Moreover, it is found that the probability distribution for all returns is well consistent with a Lorentz distribution. Since it supports to carry out the dynamical behavior in our stock and foreign exchange markets, our analysis would assure that it is able to capture the essential multifractal properties in our present result. In future, our result will be applied to extensively investigate the other tick data in Korean financial markets and compared with other calculations transacted in other nations in detail.

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FIGURE CAPTIONS

Fig. 1. Plot of the tick data for the yen-dollar exchange rate, where one time step is the transaction time evoluted for one day. This continuous tick data were taken from January 1971 to June 2003.

Fig. 2. Log-log plot of $R/S(\tau)$ at $\tau = 1$ for the yen-dollar exchange rate.

Fig. 3. Plot of The q-th price-price correlation function $F_q(\tau)$ of the time interval τ for the KOSPI, where the value of slopes is summarized in Table 1.

Fig. 4. The probability distribution of returns for the KOSPI. the dot line is represented in terms of a Lorentz distribution, i.e. $P(r) = \frac{2b}{\pi}\frac{a}{r^2+a^2}$, where $a = 7.0 \times 10^{-3}$ and $b = 9.0 \times 10^{-3}$ for the yen-dollar exchange rate.

Fig. 5. The probability distribution of all returns for the yen-dollar exchange rate. The dashed and solid lines show the Gaussian and Lorentz distributions, respectively, where the right(left) tail denotes the postive (negative) returns region.

TABLE CAPTIONS

Table 1. Summary of values of the Hurst exponent $H(\tau)$ and the generalized qth-order Hurst exponent H_q for the yen-dollar exchange rate(YDER), th won-dollar exchange rate(WDER).

	$H(\tau)$	H_q	
YDER	$H(\tau=1)=0.6513$	H_1 =0.6216	H_4 =0.4148
	$H(\tau=15)=0.5710$	H_2 =0.5688	H_5 =0.3486
	$H(\tau=25)=0.5885$	H_3 =0.4956	H_6 =0.2996
WDER	$H(\tau=1)=0.6886$	H_1 =0.6535	H_4 =0.4307
	$H(\tau=5)=0.7283$	H_2 =0.5614	H_5 =0.3914
	$H(\tau=25)=0.7372$	H_3 =0.4859	H_6 =0.3629